65/4/1

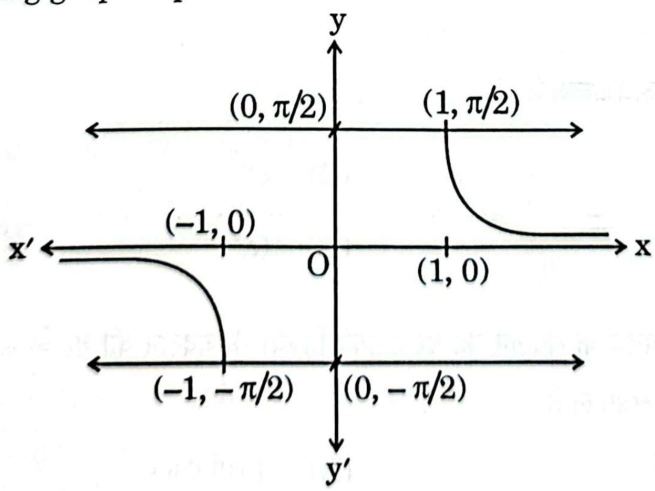
Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Certificate Examination, 2026
MATHEMATICS (041) (PAPER CODE 65/4/1)

General Instructions: -

1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4.	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9.	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

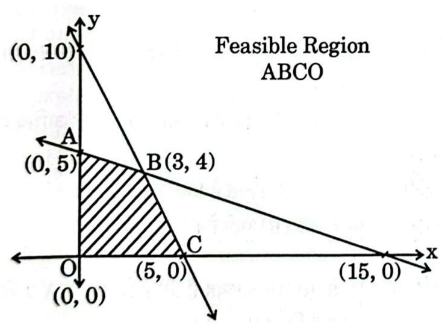
11.	A full scale of marks __________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
15.	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS CLASS XII (Subject Code–041)
(PAPER CODE: 65/4/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	The following graph represents :  (A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$		
Ans.	$(D)\ y = \operatorname{cosec}^{-1} x$		1
2.	If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \frac{\lvert i - 3j \rvert}{2}$, then A' is : (A) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{5}{2} & 1 \end{bmatrix}$		
Ans.	$(B) \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$		1

3.	**The principal value of $\sec^{-1}\left(\sqrt{2}\right) + 2\,\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is :** **(A)** $-\frac{\pi}{2}$ (B) $-\frac{\pi}{4}$ **(C)** $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$		
Ans	$(B)\ \frac{-\pi}{4}$		1
4.	If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is : (A) $\frac{4}{7}$ (B) $\frac{-3}{7}$ (C) 4 (D) $-$ 4		
Ans.	(C) 4		1
5.	Differential of e^{e^x} with respect to x is : (A) $\log\ x$ (B) e^{e^x} (C) $e^x.e^{e^x}$ (D) $(e^x)^2$		
Ans.	$(C)\ e^x.e^{e^x}$		1
6.	**The surface area of a sphere when its volume changes at the same rate as its radius is :** **(A)** **4π sq. units** (B) 1 sq. unit **(C)** **4 sq. units** (D) π sq. units		
Ans.	(B) 1 sq. units		1
7.	If $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is: (A) 0 (B) $-$ 2 (C) -1 (D) 2		
Ans.	(D) 2		1

8.	The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is ***not*** differentiable at how many points ? (A) At only one point (B) At only two points (C) At no point (D) At three points		
Ans.	(B) At only two points		1
9.	$\int \frac{dx}{\sqrt{25-16x^2}}$ is equal to : (A) $\frac{1}{5}\sin^{-1}4x+c$ (B) $\frac{1}{25}\sin^{-1}16x+c$ (C) $\frac{1}{4}\sin^{-1}\frac{4x}{5}+c$ (D) $\frac{1}{16}\sin^{-1}\frac{4x}{5}+c$		
Ans.	$(C)\ \frac{1}{4}\sin^{-1}\left(\frac{4x}{5}\right)+c$		1
10.	If $\int_0^1 \frac{dx}{e^x+e^{-x}} = \tan^{-1} e + k$, then the value of k is : (A) e (B) $\frac{\pi}{4}$ (C) 0 (D) $-\frac{\pi}{4}$		
Ans.	$(D)\ \frac{-\pi}{4}$		1
11.	The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is : (A) 2 sq. units (B) $\frac{1}{2}$sq. unit (C) 1 sq. unit (D) 4 sq. units		
Ans.	(A) 2 sq. units		1
12.	Product of the order and degree of differential equation $1+\left(\frac{dy}{dx}\right)^3 = \lambda\left(\frac{d^3y}{dx^3}\right)^2$ is : (A) 5 (B) 6 (C) 2 (D) 3		
Ans.	(B) 6		1

13.	Which of the following is **not** a Linear Differential Equation? $(A)\ \left(1+x^2\right)dy+2xydx=\cot xdx$ $(B)\ y+\frac{d}{dx}(xy)=x\left(\sin x+\log x\right)$ $(C)\ x\left(1+y^2\right)dx-y\left(1+x^2\right)dy=0$ $(D)\ ydx-\left(x+3y^2\right)dy=0$		
Ans.	$(C)\ x\left(1+y^2\right)dx-y\left(1+x^2\right)dy=0$		1
14.	The region represented by the system of inequations $3x+y\geq 3$, $2x-y\geq -5$, $x, y\geq 0$ is : (A) unbounded in 1^{st} quadrant (B) bounded in 1^{st} quadrant (C) unbounded in 2^{nd} quadrant (D) bounded in 2^{nd} quadrant		
Ans.	(A) unbounded in 1^{st} Quadrant		1
15.	In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z=px+qy,\ p,q>0$ is shaded. If all points on segment AB give max (Z), then which of the following is true?  $(A)\ p=2q$ (B) $p=3q$ $(C)\ q=3p$ (D) $q=2p$		
Ans.	$(C)\ q=3p$		1
16.	If $(3\hat{i}-2\hat{j}+5\hat{k})\times(4\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$, then the values of p and q are : (A) $p=-\frac{2}{3}, q=\frac{5}{3}$ (B) $p=-\frac{8}{3}, q=\frac{20}{3}$ (C) $p=\frac{20}{3}, q=-\frac{8}{3}$ (D) $p=0, q=0$		
Ans.	$(B)\ p=\frac{-8}{3}, q=\frac{20}{3}$		1

17.	Three points A(0, 1, 1), B(2, 0, −1) and C(1, 0, 3) form ΔABC. The ar (Δ ABC) is : (A) $\frac{\sqrt{53}}{2}$ sq. units (B) $\sqrt{53}$ sq. units (C) $\frac{\sqrt{11}}{2}$ sq. units (D) $\sqrt{11}$ sq. units		
Ans.	$(A)\ \frac{\sqrt{53}}{2}$ sq. units		1
18.	A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red ? (A) $\frac{27}{125}$ (B) $\frac{8}{125}$ (C) $\frac{2}{125}$ (D) $\frac{98}{125}$		
Ans.	$(D)\ \frac{98}{125}$		1
19.	Assertion (A): If A and B are two square matrices such that AB and BA are defined, then it is not necessary that AB = BA. Reason (R): Product of two diagonal matrices of same order is commutative.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
20.	Assertion (A): A function $f:\mathbb{N}\rightarrow\mathbb{N}$ given by $f(x)=x^3+2, \forall\ x\in\mathbb{N}$ is one-one but not onto. Reason (R): Since $\forall\ y\in\mathbb{N}$ (codomain), there does not exist $x=(y-2)^{\frac{1}{3}}$ in $\mathbb{N}$ (Domain) such that $f(x)=x^3+2=y$.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
	SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.		

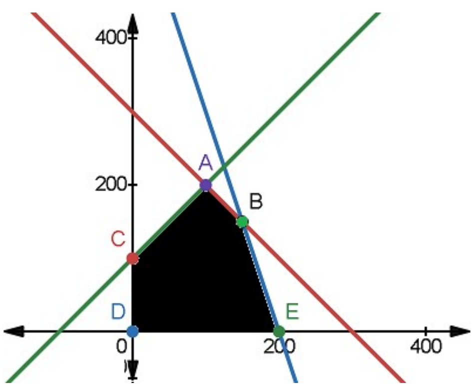
21.	Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)+\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$		2
Sol.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)+\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}-\frac{\pi}{3}$ $=\frac{-5\pi}{12}$	I II	1½ (For any 3 correct result) ½
22. (a)	Show that the function $f(x)=\begin{cases}\dfrac{\cos x}{-x+\frac{\pi}{2}}, & x\neq\frac{\pi}{2}\\ 1, & x=\frac{\pi}{2}\end{cases}$ is continous at $x=\frac{\pi}{2}$.		
Sol.	For showing: $\lim_{x\to\frac{\pi}{2}} f(x)=1$ For showing: $f\left(\frac{\pi}{2}\right)=1$ $\therefore f(x)$ is continuous at $x=\frac{\pi}{2}$.	I II III	1 ½ ½
	OR		
22. (b)	**Find whether the function f(x) = $\begin{cases}x-1, & x<2\\ 2x-3, & x\geq 2\end{cases}$ at x = 2 is differentiable or not.**		
Sol.	Getting $LHD=1$ Getting $RHD=2$ $\therefore LHD\neq RHD$, so $f(x)$ is not differentiable at $x=2$.	I II III	1 ½ ½

23.	Find the values of x for which $f(x)=x^x, x>0$ is increasing.		
Sol.	$f'(x)=x^x(1+\log x)$ Put, $f'(x)=0 \Rightarrow x^x(1+\log x)=0$ As $x^x>0$ for $x>0, (1+\log x)=0 \Rightarrow x=\frac{1}{e}$ $\therefore f(x)$ is increasing when $x>\frac{1}{e}$ or $x\in\left(\frac{1}{e},\infty\right)$ or $x\in\left[\frac{1}{e},\infty\right)$	I II III	1 ½ ½
24.	**If the position vectors of three points A, B and C are $3\hat{i}+\hat{j}$, $5\hat{i}+6\hat{j}-3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.**		
Sol.	$\overrightarrow{AB}=2\hat{i}+5\hat{j}-3\hat{k}, \overrightarrow{BC}=-5\hat{i}-2\hat{j}+3\hat{k}, \overrightarrow{CA}=3\hat{i}-3\hat{j}$ Now, $\lvert\overrightarrow{AB}\rvert=\lvert\overrightarrow{BC}\rvert=\sqrt{38}$ units, $\lvert\overrightarrow{CA}\rvert=\sqrt{18}$ units As, $\lvert\overrightarrow{AB}\rvert=\lvert\overrightarrow{BC}\rvert$ & sum of two sides is greater than the third so, given points form an isosceles triangle.	I II III	1 ½ ½
25. (a)	**Let two rods placed on the ground be represented by vectors $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.**		
Sol.	Let $\vec{a}=4\hat{i}-\hat{j}+3\hat{k}, \vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$ $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 3\\ -2 & 1 & -2\end{vmatrix}=-\hat{i}+2\hat{j}+2\hat{k}$ Now, Required vector $=5\left(\frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}\right)=\begin{cases}\frac{-5}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}\\ or\\ \frac{5}{3}\hat{i}-\frac{10}{3}\hat{j}-\frac{10}{3}\hat{k}\end{cases}$	I II	1 1 (For any one result)

	Alternative approach: Let $\vec{a}=4\hat{i}-\hat{j}+3\hat{k}, \vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$ Let required vector be $\vec{p}=x\hat{i}+y\hat{j}+z\hat{k}$ so, $25=x^2+y^2+z^2, 4x-y+3z=0, -2x+y-2z=0$ on solving we get, $z=-2x,\ y=z \Rightarrow \frac{100}{9}=z^2 \Rightarrow z=\pm\frac{10}{3}$ $\therefore$ Required vector, $\vec{p}=\begin{cases}\frac{-5}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}\\ \quad or \\ \frac{5}{3}\hat{i}-\frac{10}{3}\hat{j}-\frac{10}{3}\hat{k}\end{cases}$	I II	1 1 (For any one result)
OR			
25. (b)	**A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.**		
Sol.	$\cos^2\left(\frac{\pi}{4}\right)+\cos^2\left(\frac{\pi}{3}\right)+\cos^2\theta=1 \Rightarrow \cos\theta=\frac{1}{2}$ (As, θ is acute) $\Rightarrow \theta=\frac{\pi}{3}$ & Components of $\vec{a}=\frac{1}{\sqrt{2}},\frac{1}{2},\frac{1}{2}$	I II	1 1
	SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.		
26. (a)	Let $A=\mathbb{R}-\{3\}$ and $B=\mathbb{R}-\{1\}$. A function $f: A \to B$ is defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.		
Sol:	Let $f(a)=f(b)$ for some $a,b \in \mathbb{R}-\{3\}$ so, $\left(\frac{a-2}{a-3}\right)=\left(\frac{b-2}{b-3}\right) \Rightarrow ab-2b-3a+6=ab-3b-2a+6$ $\Rightarrow a=b$, Thus f is one-one function.	I II	1 ½

	Now, let us assume for some $y \in \mathbb{R}-\{1\},\ y=f(x)=\dfrac{x-2}{x-3}$ For getting, $x=\dfrac{3y-2}{y-1}$ Range $=\mathbb{R}-\{1\}=$ co-domain, Thus f is onto function.	III IV	1 ½
OR			
26. (b)	Let n be a fixed positive integer. A relation R is defined in set $\mathbb{Z}$ such that $R=\{(x,y):(x-y)$ is divisible by n, $x,y\in\mathbb{Z}\}$. Determine if R is an equivalence relation.		
Sol:	Let for some $a\in\mathbb{Z},(a,a)\in R$ $\Rightarrow$ 0 is divisible by n, which is true $\therefore (a,a)\in R,$ so R is reflexive. Let $(a,b)\in R \Rightarrow a-b=np,\ p\in\mathbb{Z}$ $\Rightarrow b-a=n(-p)=nq, q\in\mathbb{Z}$ $\Rightarrow b-a$ is also divisble by n. $\therefore (\mathrm{b},\mathrm{a})\in$ R, so R is symmetric. Let $(a,b)\in R,(b,c)\in R$ for some $a,b,c\in\mathbb{Z}$ $\Rightarrow a-b=np$ and $b-c=nr,$ where $p,r\in\mathbb{Z}$ $\Rightarrow a-b+b-c=n(p+r)\Rightarrow (a-c)$ is divisble by n. $\therefore (a,c)\in$ R, so R is transitive. Thus, R is an equivalence relation.	I II III IV	½ 1 1 ½
27.	If $A=\begin{bmatrix}3&2&0\\1&4&0\\0&0&5\end{bmatrix}$, then compute $A^2-7A+10I$.		

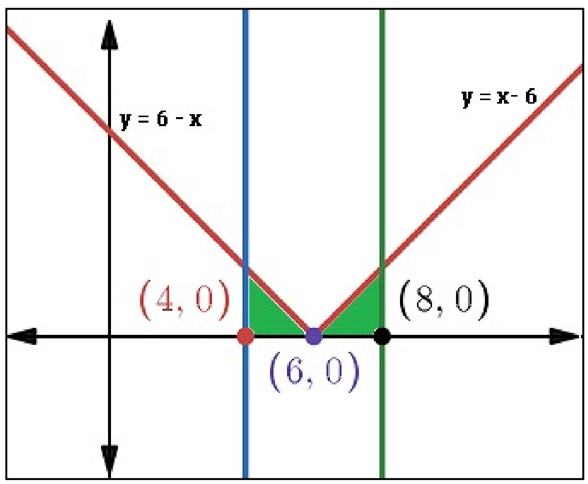
Sol.	$A^2 = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix}$ $A^2 - 7A + 10I = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$ $A^2 - 7A + 10I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	I II III	1½ (½ for any three correct elements) 1 ½
28. (a)	If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.		
Sol.	$xy = e^{x-y} \Rightarrow x\frac{dy}{dx} + y = e^{x-y}\left(1 - \frac{dy}{dx}\right)$ $\Rightarrow \left(x + e^{x-y}\right)\frac{dy}{dx} = e^{x-y} - y \Rightarrow \frac{dy}{dx} = \frac{e^{x-y} - y}{x + e^{x-y}}$ **or** $\frac{y(x-1)}{x(y+1)}$ Alternative method: $xy = e^{x-y} \Rightarrow \log x + \log y = (x - y)$ $\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{1 - \frac{1}{x}}{\frac{1}{y} + 1} = \frac{y(x-1)}{x(y+1)}$	I II I II III	1½ 1½ 1 1 1
OR			
28. (b)	Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.		

Sol.	Let $y=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ & put $x^2=\cos\theta$ $y=\tan^{-1}\left(\dfrac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$ $y=\tan^{-1}\left(\dfrac{\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$ Thus, $y=\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\Rightarrow\frac{dy}{d\theta}=\frac{-1}{2}$	 I II III	 ½ 1½ 1
29.	Solve the following Linear programming problem graphically: Maximize $Z=200x+120y$ subject to the constraints $x+y\le 300$ $3x+y\le 600$ $x-y\ge -100$ $x\ge 0, y\ge 0$		
Sol.	 Corner point / Value of Z=200x+120y: A (100,200) 44000; B (150,150) 48000; C (0, 100) 12000; D (0, 0) 0; E (200, 0) 40000 Maximum value of Z is 48000 at x = 150 & y = 150.	I II III	1 ½ **For correct graph & shading** 1 ½
30. (a)	Find a point on the line $\dfrac{x-2}{3}=\dfrac{1-y}{2}=\dfrac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point (1, 2, 3).		

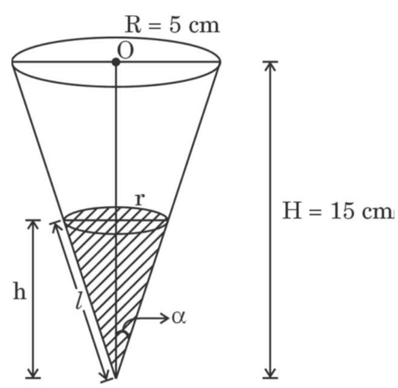
Corner point	Value of Z=200x+120y
A (100,200)	44000
B (150,150)	48000
C (0, 100)	12000
D (0, 0)	0
E (200, 0)	40000

Sol.	Let $\frac{x-2}{3}=\frac{y-1}{-2}=\frac{z-3}{2}=k$ A general point P on the line be $(3k+2,-2k+1,2k+3)$ It is given that $PQ=\sqrt{2}$ where Q(1,2,3), so $PQ=\sqrt{(3k+1)^2+(-2k-1)^2+(2k)^2}=\sqrt{2}$ $\Rightarrow 17k^2+10k=0$ $\therefore k=0$ or $\frac{-10}{17}$ Thus, required points are $(2,1,3)\ \&\ \left(\frac{4}{17},\frac{37}{17},\frac{31}{17}\right)$	I II III IV	1 1 ½ ½
OR			
30. (b)	Find the shortest distance between the lines: $\vec{r}=(4+\lambda)\hat{i}+(2\lambda-1)\hat{j}-3\lambda\hat{k}$ $\vec{r}=(1+2\mu)\hat{i}+(2-\mu)\hat{k}+(4\mu-1)\hat{j}$		
Sol:	Here, $\overrightarrow{a_1}=4\hat{i}-\hat{j},\overrightarrow{b_1}=\hat{i}+2\hat{j}-3\hat{k},\overrightarrow{a_2}=\hat{i}-\hat{j}+2\hat{k},\overrightarrow{b_2}=2\hat{i}+4\hat{j}-5\hat{k}$ Now, $\overrightarrow{a_2}-\overrightarrow{a_1}=-3\hat{i}+2\hat{k},\ \ \overrightarrow{b_1}\times\overrightarrow{b_2}=2\hat{i}-\hat{j}$ and $\left\|\overrightarrow{b_1}\times\overrightarrow{b_2}\right\|=\sqrt{5}$ Required distance $=\left\|\frac{(-3\hat{i}+2\hat{k})\bullet(2\hat{i}-\hat{j})}{\sqrt{5}}\right\|=\frac{6}{\sqrt{5}}$ or $\frac{6\sqrt{5}}{5}$	I II III	1 1½ ½
31.	In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams,		

	then find the probability that one will be a girl and the other a boy.		
Sol.	Let D be the event a debate competition is held, Q be the event a quiz competition is held and E be the event that one girl & one boy are chosen. $P(E\mid D)=2\times\frac{4}{10}\times\frac{6}{9}=\frac{8}{15}, P(E\mid Q)=2\times\frac{7}{10}\times\frac{3}{9}=\frac{7}{15}$ Now, $P(E)=\frac{1}{3}\times\frac{8}{15}+\frac{2}{3}\times\frac{7}{15}=\frac{22}{45}$	I II	1+1 1
	SECTION D Q. Numbers 32 to 35 are long answer questions of 5 marks each.		
32. (a)	Find : $\int\frac{x}{(x-1)(x^2+4)}dx$		
Sol.	Let $\frac{x}{(x-1)(x^2+4)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+4}$ on solving we get, $A=\frac{1}{5}, B=\frac{-1}{5}, C=\frac{4}{5}$ so, $\int\frac{x}{(x-1)(x^2+4)}dx=\frac{1}{5}\int\frac{dx}{x-1}-\frac{1}{10}\int\frac{2xdx}{x^2+4}+\frac{4}{5}\int\frac{dx}{x^2+4}$ $\int\frac{x}{(x-1)(x^2+4)}dx=\frac{1}{5}\log\lvert x-1\rvert-\frac{1}{10}\log(x^2+4)+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right)+c$	I II III	1 1½ 2½
	OR		
32. (b)	Evaluate : $\int_0^1\frac{x\tan^{-1}x}{(1+x^2)^{\frac{3}{2}}}dx.$		

Sol.	Let $x = \tan t \Rightarrow dx = \sec^2 t dt$ $I(\text{Let}) = \int_0^1 \frac{x\tan^{-1} x}{\left(1+x^2\right)^{\frac{3}{2}}} dx = \int_0^{\frac{\pi}{4}} \frac{t\tan t}{\sec^3 t} \sec^2 t dt = \int_0^{\frac{\pi}{4}} t\sin t dt$ $I = \left(-t\cos t + \sin t\right)_0^{\frac{\pi}{4}}$ $I = \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$ or $\frac{-\pi+4}{4\sqrt{2}}$	I II III IV	1 2 1 1
33.	Using integration, find the area of the region enclosed by the curve $y = \lvert x-6 \rvert$, the x-axis, and between $x = 4$ and $x = 8$.		
Sol.	 Required Area $= \int_4^8 y dx = \int_4^8 \lvert x-6 \rvert dx$ Area $= -\int_4^6 (x-6) dx + \int_6^8 (x-6) dx$ Area $= -\left(\frac{(x-6)^2}{2}\right)_4^6 + \left(\frac{(x-6)^2}{2}\right)_6^8 = 2+2 = 4$ or 4 sq. units	I II III IV	1 (For correct graph) 1 1 2
34. (a)	Solve the differential equation: $ye^y dx = (y^3 + 2xe^y) dy$, when y(0) = 1.		

Sol.	Given differential eq. can be written as $\dfrac{dx}{dy}-\dfrac{2}{y}x=y^2e^{-y}$	I	1
	Integrating factor $=e^{\int\frac{-2}{y}dy}=\dfrac{1}{y^2}$	II	1
	General solution: $x\left(\dfrac{1}{y^2}\right)=\int\dfrac{1}{y^2}.y^2e^{-y}dy$	III	1
	$x\left(\dfrac{1}{y^2}\right)=-e^{-y}+c$ or $x=y^2\left(-e^{-y}+c\right)$	IV	1
	Now, when $x=0, y=1: 0=-e^{-1}+c\Rightarrow c=e^{-1}$	V	½
	Required particular solution is, $x=y^2\left(e^{-1}-e^{-y}\right)$	VI	½
	OR		
34. (b)	Find the general solution of the differential equation: $(x^3-3xy^2)dx=(y^3-3x^2y)\ dy.$		
Sol.	$\dfrac{dy}{dx}=\dfrac{x^3-3xy^2}{y^3-3x^2y}$	I	1
	Put $y=vx\Rightarrow\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$, so	II	1
	$v+x\dfrac{dv}{dx}=\dfrac{1-3v^2}{v^3-3v}\Rightarrow x\dfrac{dv}{dx}=\dfrac{1-3v^2}{v^3-3v}-v=\dfrac{1-v^4}{v^3-3v}$		
	$\Rightarrow\int\dfrac{3v-v^3}{v^4-1}dv=\int\dfrac{1}{x}dx$	III	1
	$\Rightarrow\dfrac{3}{4}\log\left\|\dfrac{v^2-1}{v^2+1}\right\|-\dfrac{1}{4}\log\left\|v^4-1\right\|=\log x+c$	IV	1½
	$\therefore\dfrac{3}{4}\log\left\|\dfrac{y^2-x^2}{y^2+x^2}\right\|-\dfrac{1}{4}\log\left\|\dfrac{y^4-x^4}{x^4}\right\|=\log x+c$	V	½

35.	Find the equation of a line (in vector and cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ and is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.		
Sol.	Let $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=a$ & $\frac{x-4}{5}=\frac{y-1}{2}=z=b$ So any general point on these are $P(2a+1, 3a+2, 4a+3)$ and $Q(5b+4, 2b+1, b)$ respectively. For intersection points: $2a+1=5b+4, 4a+3=b$ on solving we get, $a=b=-1$ Thus, required point of intesection is $(-1,-1,-1)$. Required equation of line (cartesian form): $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z+1}{-8}$ Required equation of line (Vector form): $\vec{r}=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}-8\hat{k}\right)$	I II III IV V	1 1 1 1 1
	SECTION E This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.		
36.	At a birthday party, children are being served orange juice in conical cups, as shown in the figure.  		

	Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0\cdot1\ \text{cm}^3/\text{s}$. On the basis of the above information, answer the following questions: (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? OR (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.		
Sol.	(i) $\tan\alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$ (ii) Volume, $V = \frac{1}{27}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi}$ cm/sec (iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$ $\frac{dA}{dt} = \frac{2\pi h}{9}\frac{dh}{dt} = \frac{1}{30}\text{cm}^2/\text{sec}$ OR (iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10}\pi h^2}{9}$ $\frac{dS}{dt} = \frac{2\sqrt{10}\pi h}{9}\frac{dh}{dt} = \frac{\sqrt{10}}{30}\text{cm}^2/\text{sec}$	I I I II I II	1 1 1 1 1 1
37.	A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length. On the basis of the above information, answer the following questions:		

	(i) Write the equations representing the various dimensions & express them as the matrix equation $AX = B$. (ii) Find if A^{-1} exists. Justify your answer. (iii) (a) Find A^{-1}. OR (iii) (b) Find $A^2 + 7I$.		
Sol.	(i) $\underset{A}{\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix}} \underset{X}{\begin{pmatrix} l \\ b \\ h \end{pmatrix}} \underset{=}{=} \underset{B}{\begin{pmatrix} 3 \\ 10 \\ -1 \end{pmatrix}}$	I	1
	(ii) $\|A\| = -8 \neq 0,$ so A^{-1} exists.	I	1
	(iii) (a) $A^{-1} = \frac{1}{\|A\|}(\text{adjA}) = \frac{1}{-8}\begin{pmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{pmatrix}$ OR	I	2 (1 Mark for any 4 correct entries)
	(iii) (b) $A^2 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{pmatrix}$	I	1½
	Thus, $A^2 + 7I = \begin{pmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{pmatrix}$	II	½
	Alternative : If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$, then $\|A\| = 8$ & $A^{-1} = \frac{1}{8}\begin{pmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{pmatrix}$		1 + 2
	$A^2 = \begin{pmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{pmatrix}$, $A^2 + 7I = \begin{pmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{pmatrix}$	I II	1 1
	Note: In this question, students may take different A by rearranging of terms so check the correctness of the solution accordingly.		

38.	An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, & Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where i = 1, 2, 3. Based on the above information, answer the following questions : A person selects a cap. (i) What is the probability that he selects a red cap ? (ii) If he selects a green cap, what is the probability that the cap has come from Box II ?		
Sol.	(i) Let B_1, B_2 & B_3 be the event that box I, box II and box III selected respectively. $P(B_1)=\frac{1}{6}, P(B_2)=\frac{2}{6}, P(B_3)=\frac{3}{6}$ Now, P(selecting a red cap) $=\frac{1}{6}\times 1+\frac{2}{6}\times\frac{2}{3}+\frac{3}{6}\times 0=\frac{7}{18}$ (ii) P(selecting a green cap) $=1-\frac{7}{18}=\frac{11}{18}$ Now, $P\left(\frac{B_2}{\text{Green cap}}\right)=\frac{\frac{2}{6}\times\frac{1}{3}}{\frac{11}{18}}=\frac{2}{11}$	I II I II	1 1 1 1